ISEG – Lisbon School of Economics and Management
ECONOMETRICS
First Semester 2017/2018
Problem Set V – Time Series (II)

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	4	4	4	8	30	50

Justify all your answers (except for multiple choice questions). You are required to show your work on each problem (except for multiple choice questions). **Organize your work**. Work scattered all over the page will receive very little credit. A correct answer in a multiple choice question worths 4 points; an incorrect one worths -1 point.
Please, submit the assignment by email latest the 19th of December 2017, 11 am.

(4) **1**. Which of the following is TRUE.

- ◯ A process is highly persistent because is not constant on time.
- ◯ A weakly dependent process cannot have a trend on time.
- √ **A random walk process has a variance that depends on time.**
- ◯ None of the previous statements is correct.

(4) **2**. Consider the following model $y_t = e_t - 0.25e_{t-2}$ with e_t a white noise process with mean zero and variance σ^2. Then, one can conclude that,

- ◯ $Var(y_t) = 0.75\sigma^2$
- ◯ $Cov(y_t, y_{t+1}) = -0.25\sigma^2$ and $Cov(y_t, y_{t+2}) = 0$
- √ $Cov(y_t, y_{t+1}) = 0$ **and** $Cov(y_t, y_{t+2}) = -0.25\sigma^2$
- ◯ $Cov(y_t, y_{t+2}) = -0.25$

(4) **3**. Which of the following is FALSE.

- ◯ A serially correlated process may be weakly dependent.
- √ **A process is I(1) if it is stationary of order 1 and weakly dependent.**
- ◯ A random walk is not weakly dependent.
- ◯ A random walk with a drift is not covariance stationary.

4. With a sample of 75 annual observations the following equations were estimated:

$$\hat{y}_t = 0.15 + 0.26x_t - 0.11x_{t-1} \quad R^2 = 0.721 \quad (1)$$

$$\widehat{\hat{u}_t} = -0.021 + 0.007x_t + -0.001x_{t-1} + 0.358\hat{u}_{t-1} \quad R^2 = 0.167$$

(5) (a) The second equation has the aim to perform a test. Identify that test, write the test hypothesis, and conclude.

Solution: The second equation is used to perform a serial correlation test, in this case of order 1. It corresponds to the estimation of the equation:

$$\hat{u}_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \rho\hat{u}_{t-1} + v_t$$

Our null hypothesis is:

$$\text{H0: } \rho = 0 \text{ vs } \quad \text{H1: } \rho \neq 0$$

We can perform a Breusch-Godfrey test, using the test-statistic:

$$LM = (n-q)\, R^2_{\hat{u}^2} \xrightarrow{d} \chi^2_q$$

Since $q = 1$:

$$LM = 74 \times 0.167 = 12.358$$

Considering $\alpha = 5\%$, the critical value for a chi-squared distribution with 1 degree of freedom is 3.84 - we reject the null hypothesis, meaning there is evidence of serial correlation of order 1.

(3) (b) Is model (1) dynamically complete? Justify.

Solution: Since there is evidence of serial correlation, as concluded in a), the model can not be dinamically complete.

5. To explain the logarithm of CO2 emissions, LCO2, as a function of the logarithm of GDP, LGDP, the model in equation 1 in the Annex was estimated.

(7.5) (a) Interpret the estimates of the coefficients associated to LGDP and to the trend.

Solution:

$$lco2_t = \beta_0 + \beta_1\, t + \beta_2\, lco2_{t-1} + \beta_3\, lgdp_t + u_t$$

$\hat{\beta}_1$: ceteris paribus, the amount of CO2 emissions drops in average by $0.016883 \times 100 = 1.6883\%$ every year.

$\hat{\beta}_3$: ceteris paribus, a raise of the GDP by 1% raises the amount of CO2 emissions by 1.289%.

(7.5) (b) What was the aim of estimating Equation 2? What can you conclude from the results in that equation?

Solution: The aim of estimating equation 2 is to perform a serial correlation test of order 1. It uses the regression:

$$\hat{u}_t = \alpha_0 + \alpha_1 lco2_{t-1} + \alpha_2 lgdp_t + \rho \hat{u}_{t-1} + v_t$$

We should not use the Durbin-Watson statistic, as this model is AR(1) - it does not satisfy the strict exogeneity assumption. The Breusch-Godfrey test is more suitable for time series models; we use the test-statistic:

$$\text{H0: } \rho = 0 \text{ vs } \quad \text{H1: } \rho \neq 0$$

$$LM = (n-q)\, R^2_{\hat{u}^2} \xrightarrow{d} \chi^2_1$$

$$LM = 31 \times 0.054527 = 1.690337$$

The critical value is 3.84: we do not reject the null hypothesis - there is no evidence of serial correlation of order 1.

(7.5) (c) Write the condition that states that the model estimated in Equation 1 is dynamically complete.

Solution:

$$E[lco2_t | lgdp_t, lco2_{t-1}, lgdp_{t-1}, lco2_{t-2}, ...] = E[lco2_t | lgdp_t, lco2_{t-1}]$$

(7.5) (d) Assume that LGDP is sequentially exogenous. Moreover, suppose that $corr(LCO2_t, LCO2_{t-1}) = 0.28$ and that $corr(LGDP_t, LGDP_{t-1}) = 0.53$. Discuss the verification in Equation 1 of the assumptions needed to state the asymptotic properties of OLS.

Solution: Concerning TS.1' the model is linear in the coefficients and it is expected that LCO2 and LGDP be, most probably, weak dependent given that the respective first order correlation coefficient is very apart from 1. Therefore, TS.1' is expected to be verified. On the other hand, if LGDP is sequentially exogenous, then it is also contemporaneously exogenous, which means assumption TS.3' is verified. There is no evidence of serial correlation, as we found in b) - TS.5 is also verified. These two are usually the most commonly not held: assuming all errors are heteroskedastic, we can conclude that all necessary assumptions (TS.1', TS.2', TS.3', TS.4', TS5) hold - the asymptotic properties of OLS are valid, that is. OLS is consistent and asymptotically most efficient.